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Flatons and Peccei-Quinn Symmetry

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Abstract

We study in detail a supersymmetric Peccei-Quinn model, which has a DFSZ and a KSVZ version. The fields breaking the Peccei-Quinn (PQ) symmetry correspond to flat directions (flaton fields) and have unsuppressed couplings when PQ symmetry is unbroken. The models have interesting particle physics phenomenology. The PQ scale is naturally generated through radiative corrections; also, in the DFSZ case, the μ problem can be solved and neutrino masses can be generated. Cosmologically they lead to a short period of thermal inflation making the axion an excellent dark matter candidate if one of the flaton fields has a positive effective mass-squared at early times but with too low a density in the opposite case. A highly relativistic population of axions is produced by flaton decay during the subsequent reheating, whose density is constrained by nucleosynthesis. We compute all of the relevant reaction rates and evaluate the nucleosynthesis constraint. We find that the KSVZ model is practically ruled out, while the DFSZ model has a sizable allowed region of parameter space.

1   Introduction

With the discovery of the instantons it was realized that the pure gradient topological term of the QCD Lagrangian $\theta_{QCD}g_S^2/32\pi^2 F\tilde{F}$ can generate important physical consequences. In fact the induced CP violation affects the electric dipole moment of the neutron suggesting the limit $\theta_{QCD} \leq 10^{-10}$. The most attractive explanation for the origin of such a small parameter would be the Peccei-Quinn mechanism [1]. There is supposed to be a spontaneously broken global $U(1)$ symmetry (PQ symmetry), which is also explicitly broken by the color anomaly. The corresponding pseudo-Goldstone boson is called the axion [2].

   The PQ symmetry acts on some set of fields ϕ_i with charges Q_i,

$$\phi_i \rightarrow e^{iQ_i\alpha}\phi_i \,. \tag{1}$$

To generate the require $U(1)_{PQ} \times SU(3)_c \times SU(3)_c$ anomaly we must choose appropriate particle spectra. Depending on the charge of the SM matter content, we can have the KSVZ (hadronic) models [3] in which only some extra heavy quark fields are PQ charged or the DFSZ [4] models in which, beyond the extra matter content (at least two Higgs fields), also the Standard Model (SM) matter is PQ charged.

   Denoting the vacuum expectation values of the scalar fields charged under PQ by $v_i/\sqrt{2}$, the PQ symmetry breaking scale F_{PQ} is defined as $F_{PQ}^2 = \sum_i Q_i^2 v_i^2$, and the axion mass is given by $F_{PQ}m_a = (79\,\mathrm{MeV})^2 N$ where N is the number of quarks with PQ charge. Collider and astrophysics constraints require $F_{PQ} \gtrsim 10^9\,\mathrm{GeV}$, and defining as usual $M_{Pl} = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}$ this allows the range

$$10^9\,\mathrm{GeV} \lesssim F_{PQ} \lesssim M_{Pl} \,. \tag{2}$$

With typical assumptions about the cosmology, the requirement that axions give at most critical density places F_{PQ} towards the bottom of this range. In the particular case that the axions are radiated by strings with no subsequent entropy production, one probably requires [5] $F_{PQ} \sim 10^{10}\,\mathrm{GeV}$.

   In a model with unbroken supersymmetry, the holomorphy of the superpotential ensures that PQ symmetry Eq. (1) is accompanied by a symmetry acting on the radial parts of the PQ charged fields

$$\phi_i \rightarrow e^{Q_i\alpha}\phi_i \,. \tag{3}$$

The corresponding pseudo-Goldstone boson is called the saxion (or saxino), and the spin-half partner is called the axino. Soft supersymmetry breaking gives the saxion a mass of order $100\,\mathrm{GeV}$, and the axino typically has a mass of the same order [6] though it may be very light in special cases.[1]

   Let us consider the potential of the fields ϕ_i which break PQ symmetry. In a supersymmetric model there have to be at least two, but let us pretend for the moment that there is

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[1]We assume gravity-mediated supersymmetry breaking, which typically gives soft scalar masses in the range 100 to 1000 GeV, and use the former estimate for definiteness.

only one. Its potential will be of the form

$$V = V_0 - m^2|\phi|^2 + \frac{1}{4}\lambda\phi^4 + \sum_{n=1}^{\infty} \lambda_n \frac{|\phi|^{2n+4}}{M_{\text{Pl}}^{2n}} \, . \tag{4}$$

The non-renormalizable terms are expected to have coefficients $\lambda_n \sim 1$. If the renormalizable coupling λ is also of order 1, $m \sim F_{\text{PQ}}$ and the non-renormalizable terms are negligible. (Remembering that there are at least two complex fields, only the particular combination of fields corresponding to the saxion will have the soft mass of order $100\,\text{GeV}$.) In this sort of model one can hope to understand a value $F_{\text{PQ}} \sim 10^{10}\,\text{GeV}$ since that is the supersymmetry breaking scale [7], but it may be hard to understand a bigger scale.

We are concerned with a different class of models [8, 9, 10, 11, 12], in which ϕ represents a flat direction of supersymmetry. The quartic term is then absent, while m is a soft mass of order $100\,\text{GeV}$. If the ϕ^6 term is present with unsuppressed coefficient the vev is $\langle\phi\rangle \sim \lambda_1^{-1/4}10^{10}$ GeV. If instead the ϕ^8 dominates one has $\langle\phi\rangle \sim \lambda_2^{-1/6}10^{13}$ GeV and so on. For future reference, note that the mass of $|\phi|$ in the vacuum is also of order of m, and that the height of the potential is given by

$$\left(\frac{V_0^{1/4}}{10^6\,\text{GeV}}\right) \sim \left(\frac{\langle\phi\rangle}{10^{10}\,\text{GeV}}\right)^{1/2} \, . \tag{5}$$

Fields of this kind, characterized by a large vev and a flat potential are called *flaton* fields, and the particles corresponding to them are called flatons [13].

In a supersymmetric model there are $n \geq 2$ complex fields which all acquire vevs. We are interested in the case where these are flaton fields. Then there are $2n-1$ flaton particles with mass of order $100\,\text{GeV}$, and n flatinos with typically similar masses. The saxion (axino) is a linear combination of the flaton (flatino) fields, with no particular significance.

The rest of this paper is as follows. In the next section we define our models and summarize the cosmology. In section 3, we give the general structure of the flaton and flatino masses. In section 4 we analyze the general self interactions between flatons and flatinos. In section 5 we see the effect of the interaction of the flatons with the matter fields (The KSVZ flatons interact only with gluons and gluinos whereas DFSZ flatons interact also with ordinary matter and supermatter.) In section 6 we find the parameter space regions that can satisfy the cosmological constraints. We conclude in Section 7.

2   The model and its cosmology

We consider a DFSZ and a KSVZ (hadronic) model.

In both models, there are two flaton fields P and Q, interacting with the superpotential [11, 12]

$$W_{\text{flaton}} = \frac{f}{M_{\text{Pl}}}\hat{P}^3\hat{Q} \, . \tag{6}$$

In the hadronic version, the interaction with matter is

$$W_{\text{flaton-matter}} = h_{E_i} \hat{E}_i \hat{E}_i^c \hat{P} \tag{7}$$

where E_i and E_i^c are additional heavy quark and antiquark superfields.

In the DFSZ version, the interaction is

$$W_{\text{flaton-matter}} = \frac{1}{2}\lambda\hat{N}\hat{N}\hat{P} + \frac{g}{M_{\text{Pl}}}\hat{H}_1\hat{H}_2\hat{P}\hat{Q} \tag{8}$$

where $\hat{N}$ are the right handed neutrino superfields and $\hat{H}_{1,2}$ the two Higgs doublets. Due to the second term we can provide a solution to the μ problem [14]. In such case we can add to the superpotential of the minimal supersymmetric standard model also the terms $h_\nu\hat{l}\hat{H}_2\hat{N}$ that generate the necessary mixing between left and right neutrinos to implement a see saw mechanism which can explain the solar and atmospheric neutrino deficits.

The cosmology of the DFSZ model has already been considered in a rough way [15]. Here we relatively complete treatment of both models.

To study the cosmology of the flaton fields, we can safety analyze only the superpotential W_{flaton} for both models. With the inclusion of the soft susy breaking terms, the potential is

$$V = m_P^2|\phi_{\text{P}}|^2 + m_Q^2|\phi_{\text{Q}}|^2 + \frac{f^2}{M_{\text{Pl}}^2}\left(9|\phi_{\text{P}}|^4|\phi_{\text{Q}}|^2 + |\phi_{\text{P}}|^6\right) + \left(\frac{A_f}{M_{\text{Pl}}}f\phi_{\text{P}}^3\phi_{\text{Q}} + h.c.\right). \tag{9}$$

The soft parameters m_P, m_Q and A_f are all of order $10^{2-3}\,\text{GeV}$ in magnitude. It is assumed that m_P^2 and m_Q^2 are both positive at the Planck scale. The unsuppressed interactions of ϕ_P give radiative corrections which drive m_P^2 to a negative value at the PQ scale, triggering a vev for ϕ_P. As a result, when ϕ_P gets a vev, the tadpole term proportional to A_f generates automatically a vev for the ϕ_Q field, both of them are $v_{\text{Q}} \sim v_{\text{P}} \sim 10^{10-12}\,\text{GeV}$.

In the early Universe when $H \gtrsim 100\,\text{GeV}$, there will be effective values $m_P^2(t)$ and $m_Q^2(t)$. Supergravity interactions will make both of them at least of order H^2 in magnitude.[2] The cosmology depends on the signs of these effective values. If $m_P^2(t)$ is positive, ϕ_P is held at the origin by the finite-temperature potential until $T \sim |m_{\text{P}}|$. But the potential V_0 dominates the energy density in the regime $|m_{\text{P}}| \lesssim T \lesssim V_0^{1/4}$, leading to about $\sim \ln\left(V_0^{1/4}/|m_{\text{P}}|\right) \sim 10$ e-folds of thermal inflation [13].

To discuss what happens after thermal inflation, we suppose first that $m_Q^2(t)$ is also positive in the early Universe, so that ϕ_Q is also trapped at the origin during thermal inflation. When thermal inflation ends, ϕ_P moves away from the origin, which destabilizes ϕ_Q. The fields ϕ_P and ϕ_Q move around an orbit in field space, which would be closed if there were no energy loss. If the only energy loss came from Hubble damping the fields would oscillate back and forth many times around an almost-closed orbit. However, the parameter determining the strength of parametric resonance is $q \sim g\Phi_0/m \sim 10^8$, where $g \sim 1$ is a typical coupling, $\Phi_0 \sim 10^{10}\,\text{GeV}$ is the amplitude of the oscillation and $m \sim 10^2\,\text{GeV}$ is its angular frequency. One therefore expects that parametric resonance will efficiently damp

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[2]During inflation this result might be avoided (say by D-term inflation) but it should still hold afterwards.

the orbit, converting most of the energy into flaton particles. At first these particles are marginally relativistic, but after a few Hubble times they become non-relativistic. The rest of the energy resides in the homogeneous oscillating flaton fields, now with small amplitude and therefore almost simple harmonic motion corresponding to some more non-relativistic flatons. The flatons decay leading to final reheating at a temperature [13]

$$T_{\rm RH} \simeq 1.2 g_{RH}^{-\frac{1}{4}} \sqrt{M_{\rm Pl} \Gamma_\phi} \sim 3 \left(\frac{10^{11}\,{\rm GeV}}{F_{\rm PQ}} \right) \left(\frac{m}{300\,{\rm GeV}} \right)^3 {\rm GeV}\,.$$

where m is the mass of the lightest flaton and $g_{RH} \sim 100$ is the effective number of relativistic species at reheat.

Two different axion populations are produced. One population is radiated by the PQ strings that form after thermal inflation. They become dark matter with abundance [5] $\Omega_{\rm a} \sim (F_{\rm PQ}/10^{10}\,{\rm GeV})^{1.2}$. The present scenario *predicts $F_{\rm PQ} \sim 10^{10}$ GeV, making the axion an excellent dark matter candidate.* Note that in contrast with the general case, the axion density in this scenario cannot be reduced by entropy production after the epoch $T \sim 1\,{\rm GeV}$ when the axions acquire mass; pre-existing long-lived particles that might do the job have been diluted away by the thermal inflation.

The other axion population, that is our main concern, comes from the decay of the flatons [13, 6]. This population is still relativistic at nucleosynthesis, and its density must satisfy the constraint

$$\left(\frac{\rho_a}{\rho_\nu} \right)_{NT} \leq \delta N_\nu \sim 0.1 - 1.5\,, \tag{10}$$

where ρ_ν denotes the energy density of a single species of relativistic neutrino and δN_ν the number of extra neutrino species allowed by nucleosynthesis.

If there were just one species of flaton field ϕ, this would give the bound [15]

$$\left(\frac{\rho_a}{\rho_\nu} \right)_{NT} = \frac{43}{7} \left(\frac{43/4}{g_{RH}} \right)^{1/3} \frac{B_a}{1 - B_a}$$

$$\simeq \frac{43}{7} \left(\frac{43/4}{g_{RH}} \right)^{1/3} B_a\,, \tag{11}$$

where $B_a = \Gamma_a (\phi \to a + a) / \Gamma_{tot}$ is the branching ratio, and in the last line we assumed $B_a \ll 1$. From Eq. (10) we get the bound

$$\frac{\Gamma (\phi \to a + a)}{\Gamma(\phi \to X)} \leq 0.24 \left(\frac{\delta N}{1.5} \right) \left(\frac{g_{RH}}{43/4} \right)^{1/3}\,. \tag{12}$$

Varying T_{RH} from 6 MeV to $m \sim 100\,{\rm GeV}$ we get a factor two variation coming from the number of degrees of freedom. For δN_ν varying from 0.1 to 1.5 we get $B_a < 1/3$ to $B_a < 0.02$.

Our model has three flaton particles and the quantity B_a to be used in Eq. (11) is

$$B_a = \sum_I r_I B_I \quad \text{where} \quad r_I = \frac{n_I}{\sum_J n_J}\,,$$

$$B_I \equiv \frac{\Gamma (I \to a + a) + \frac{1}{2}\Gamma (I \to a + X)}{\Gamma_{\rm tot} (I)}\,. \tag{13}$$

4

Here n_I is the number density of the Ith flaton just before reheating, related to the mass density ρ_I by $n_I = \rho_I/m_I$. In principle one could calculate the n_I (coming from parametric resonance and some residual homogeneous flaton oscillations decay) but we have not done that, and to estimate the allowed parameter range for the model we shall simply assume $r_I = 1/3$.

Next consider what happens after thermal inflation in the case that $m_Q^2(t)$ is negative. During thermal inflation, while ϕ_P is trapped at the origin, $|\phi_Q|$ will have some value $\gg F_{\text{PQ}}$ determined by a higher-order non-renormalizable term. At the end of thermal inflation, all three flaton particles will be produced in the manner we have described, and the nucleosynthesis constraint still holds. The difference from the previous case is that axionic strings are not produced, so that dark matter axions are produced only by the quantum fluctuation of the axion field during inflation. The density is now [16, 17] $\Omega_{\text{a}} \sim 3(\theta/\pi)^2(F_{\text{PQ}}/10^{12}\,\text{GeV})^{1.2}$, where $\theta < \pi$ is the misalignment angle. Again, thermal inflation means that the abundance will not be diluted by entropy production, so discounting an accidentally small θ we again have a rather definite prediction, which is too low. For the axions to be the dark matter in this scenario, one would have to increase F_{PQ} by generating it from a higher-order non-renormalizable term.

Finally, if $m_P^2(t)$ is negative in the early Universe, there is no thermal inflation and one is back with all the uncertainties of more general models [17, 18, 19]. Axion cosmology now depends on the scale of the inflaton potential, the reheat temperature after inflation, the decoupling temperature of the flatons and so on. We have nothing to say about that case.

3 Flaton and flatino spectrum

3.1 Flaton spectrum

We write the flaton fields as

$$
\begin{aligned}
\phi_{\text{P}} &= \frac{v_P + P}{\sqrt{2}} e^{i\frac{A_P}{v_P}} \\
\phi_{\text{Q}} &= \frac{v_Q + Q}{\sqrt{2}} e^{i\frac{A_Q}{v_Q}} \,,
\end{aligned}
\tag{14}
$$

and we shall take v_P, v_Q, A_f and f as the independent parameters in the potential Eq. (9). The main components of the axion field are

$$
a = -\frac{v_P}{F_{PQ}} A_P + 3\frac{v_Q}{F_{PQ}} A_Q
\tag{15}
$$

where $F_{\text{PQ}}^2 = v_P^2 + 9v_Q^2$ and we have neglected the $O\left(v_{EW}/F_{\text{PQ}}\right)$ components along the $H_{1,2}$ directions. The orthogonal field to the axion (both are CP Odd) corresponds to a flaton particle. It is

$$
\psi' = -\frac{v_P}{F_{\text{PQ}}} A_Q - 3\frac{v_Q}{F_{\text{PQ}}} A_P
\tag{16}
$$

5

which has a mass

$$M_{\psi'}^2 = -\frac{fA_fv_PF_{PQ}^2}{2M_{Pl}v_Q} = -\frac{f}{g}\mu A_f\left(x^2+9\right).$$ (17)

where

$$\frac{\mu}{g} \equiv \frac{v_Pv_Q}{2M_{Pl}}$$ (18)

$$x \equiv \frac{v_P}{v_Q}.$$ (19)

For future convenience we have introduced a quantity μ, related to the g appearing only in the DFSZ model. At this stage results depend only on the ratio μ/g defined by (18) and they apply to both models. Since $M_{\psi'}^2$ is positive, A_f and f must have opposite signs.

The other two flaton particles correspond to the CP even fields P and Q. They have a 2⊗2 mass matrix whose components are

$$M_{QQ}^2 = M_{\psi'}^2\frac{x^2}{9+x^2}$$ (20)

$$M_{PQ}^2 = 9f^2\frac{v_P^4}{M_{Pl}^2x} - 3\frac{M_{\psi'}^2x}{9+x^2} = 3x\left(12\frac{f^2}{g^2}\mu^2 - \frac{M_{\psi'}^2}{9+x^2}\right)$$

$$M_{PP}^2 = 3f^2\frac{v_P^4\left(x^2+3\right)}{M_{Pl}^2x^2} - 3\frac{M_{\psi'}^2}{9+x^2} = 12\frac{f^2}{g^2}\left(x^2+3\right)\mu^2 - 3\frac{M_{\psi'}^2}{9+x^2}.$$

Here two mass parameters m_P^2, m_Q^2 in Eq. (9) are replaced in favor of v_P, v_Q. Performing the rotation from the flavor basis $||P\ Q||$ to the mass basis $||F_1\ F_2||$

$$P = \cos\alpha F_2 - \sin\alpha F_1$$
$$Q = \sin\alpha F_2 + \cos\alpha F_1,$$ (21)

the mixing angle results

$$\tan 2\alpha = \frac{2M_{PQ}^2}{M_{QQ}^2 - M_{PP}^2} = -6\frac{x}{x^2+3} \equiv -a.$$ (22)

Note that $\tan 2\alpha$ ranges from $-\sqrt{3}$ for $x = \sqrt{3}$ to zero for $x \gg 1$ and $x \to 0$. For each x there are two solutions given by

$$\cos^2\alpha_1 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1+a^2}}\right)$$ (23)

$$\cos^2\alpha_2 = \frac{1}{2}\left(1 + \frac{1}{\sqrt{1+a^2}}\right)$$ (24)

with $\sin\alpha_{1,2} \geq 0$. The two solutions will be relevant when we discuss the parameter space for the decay of flatons into axions.

6

The two eigenstates have masses

$$M^2_{F_{2,1}} = \frac{\mu^2}{2} \left(\frac{f}{g}(12(x^2+3)\frac{f}{g} + (3-x^2)\frac{A_f}{\mu}) \pm |\frac{f}{g}(12\frac{f}{g} + \frac{A_f}{\mu})|\sqrt{9+42x^2+x^4} \right) \quad (25)$$

The requirement of a positive definite spectrum $(m^2_{F_1} > 0)$ gives the constraint

$$y_1 < y = -\frac{g}{f}\frac{A_f}{\mu}\frac{9+x^2}{4x^2} < y_2 \,, \quad (26)$$

where
$$y_{1,2} \equiv \frac{9+x^2}{8x^2}\left(21+x^2 \pm \sqrt{9+42x^2+x^4}\right) \quad (27)$$

or
$$\frac{1}{2}\left(21+x^2 - \sqrt{9+42x^2+x^4}\right) < -\frac{g}{f}\frac{A_f}{\mu} < \frac{1}{2}\left(21+x^2 + \sqrt{9+42x^2+x^4}\right) \quad (28)$$

and requiring positive diagonal elements implies also

$$y < y_3 \equiv \frac{(x^2+3)(x^2+9)}{x^2} \quad (29)$$

3.2   Flatino spectrum

From the superpotential W_{flaton} we can directly extract also the flatino's mass matrix whose eigenvalues are

$$M^2_{\tilde{F}_{2,1}} = \frac{9}{4}\frac{M^2_{\psi'}}{y\,x^2}[x^2+2\pm\sqrt{x^2+1}] = 9\frac{f^2}{g^2}\mu^2[x^2+2\pm\sqrt{x^2+1}] \quad (30)$$

The eigenstates $\tilde{F}_1, \tilde{F}_2$ are related to the flavor states $\tilde{P}, \tilde{Q}$ by

$$
\begin{aligned}
\tilde{F}_1 &= \cos\tilde{\alpha}\tilde{P} + \sin\tilde{\alpha}\tilde{Q} \\
\tilde{F}_2 &= -\sin\tilde{\alpha}\tilde{P} + \cos\tilde{\alpha}\tilde{Q}
\end{aligned}
\quad (31)
$$

where $\tan 2\tilde{\alpha} = x$,

A parameter space analysis indicates that we have always $M_{F_1} \geq 2M_{\tilde{F}_1}$. This automatically forbids the decay of F_1 to flatinos leaving open only the decay into flatinos of the heavier F_2 and ψ' flatons.

4   Interactions between flatons

Now that we know the general mass matrix structure of the flatons and flatinos, common both to the DFSZ and KSVZ models, we can start analyzing the various decay rate between

flatonic fields.[3] We begin with the decay channels induced by the kinetic term and the superpotential W_{flaton}, which are common to the KSVZ and the DFSZ models.

4.1 Derivative and cubic interaction terms between flatons

The flaton interaction terms with at least one derivative are given by the lagrangian

$$
\begin{aligned}
L_\partial &= \frac{2\,v_P\,P + P^2}{2v_P^2}\left[\frac{v_P^2}{F_{\text{PQ}}^2}\left(\partial a\right)^2 + 9\frac{v_Q^2}{F_{\text{PQ}}^2}\left(\partial\psi'\right)^2 + 6\frac{v_Q v_P}{F_{\text{PQ}}^2}\partial\psi'\partial a\right] + \\
&\quad \frac{2\,v_Q\,Q + Q^2}{2v_Q^2}\left[9\frac{v_Q^2}{F_{\text{PQ}}^2}\left(\partial a\right)^2 + \frac{v_P^2}{F_{\text{PQ}}^2}\left(\partial\psi'\right)^2 - 6\frac{v_Q v_P}{F_{\text{PQ}}^2}\partial\psi'\partial a\right]
\end{aligned}
\tag{33}
$$

From this expression we can extract the following terms expressed in mass eigenstates.

The trilinear derivative interactions with no axions

$$
\left(\partial\psi'\right)^2 \frac{1}{F_{\text{PQ}}x\sqrt{x^2+9}}\left[\left(-9\sin\alpha + x^3\cos\alpha\right)F_1 + \left(9\cos\alpha + x^3\sin\alpha\right)F_2\right],
\tag{34}
$$

The trilinear derivative interactions with only one axion

$$
L_{F_i\psi'a} = \partial\psi'\partial a\frac{6}{F_{\text{PQ}}\sqrt{x^2+9}}\left[\left(\cos\alpha - x\sin\alpha\right)F_2 - \left(\sin\alpha + x\cos\alpha\right)F_1\right],
\tag{35}
$$

The trilinear derivative interactions with two axions

$$
L_{F_iaa} = |\partial_\mu a|^2\frac{1}{F_{\text{PQ}}\sqrt{x^2+9}}\left(\left(9\cos\alpha - x\,\sin\alpha\right)F_1 + \left(9\sin\alpha + x\,\cos\alpha\right)F_2\right).
\tag{36}
$$

All the above derivative interactions can be transformed in scalar interactions if we are working at tree level and with on-shell external particles

$$
\phi_1\left(\partial_\mu\phi_2\right)\left(\partial^\mu\phi_3\right) = \frac{1}{2}\left(M_{\phi_1}^2 - M_{\phi_2}^2 - M_{\phi_3}^2\right)\phi_1\phi_2\phi_3
\tag{37}
$$

The cubic interactions come also from the flatonic superpotential and the soft terms

$$
\begin{aligned}
L_{\phi^3} &= \left(\frac{9}{2}f^2\frac{v_P v_Q^2}{M_{\text{Pl}}^2} + \frac{5}{2}f^2\frac{v_P^3}{M_{\text{Pl}}^2} - \frac{M_{\psi'}^2 v_Q x}{v_P^2\left(x^2+9\right)}\right)P^3 + \left(\frac{27}{2}f^2\frac{v_P^2 v_Q}{M_{\text{Pl}}^2} - 3\frac{M_{\psi'}^2 x}{v_P\left(x^2+9\right)}\right)P^2 Q \\
&\quad + \left(\frac{9}{2}f^2\frac{v_P^3}{M_{\text{Pl}}^2}\right)PQ^2 + \left(\frac{3f}{4}\frac{A_f F_{\text{PQ}}^2}{M_{\text{Pl}}v_Q}\right)P\psi'\psi' + \left(\frac{f}{4}\frac{A_f F_{\text{PQ}}^2 v_P}{M_{\text{Pl}}v_Q^2}\right)Q\psi'\psi'.
\end{aligned}
\tag{38}
$$

---

[3]The two body decay rate is given by the expression

$$
\Gamma\left(i \to f_1 f_2\right) = \frac{1}{16\pi}\frac{S|P|}{M_i^2}\int_{-1}^{1}d\cos\theta\, A\left(E_1 = E_2, \cos\theta\right)
\tag{32}
$$

where S=1 if $f_1 \neq f_2$ and $S = 1/2$ is $f_1 = f_2$; $E_1 = E_2 = \left(M_i^2 - M_{f_1}^2 - M_{f_2}^2\right)/2M_i$, $|P| = 1/\left(2M_i\right)\sqrt{\left(M_i^2 - (M_{f_1} + M_{f_2})^2\right)\left(M_i^2 - (M_{f_1} - M_{f_2})^2\right)}$ and $A\left(..\right)$ is the square amplitude of the decay.

8

In the mass basis, we get

$$
\begin{aligned}
L_{F_2F_1F_1} &= \left\{ -3\frac{M_{\psi'}^2 \sin\alpha}{F_{\mathrm{PQ}}x\sqrt{9+x^2}} \left(-2x\cos^2\alpha + x\sin^2\alpha + \cos\alpha\sin\alpha\right) + \right. \\
&\quad \frac{6\,f^2\,\mu^2\sqrt{x^2+9}}{g^2xF_{\mathrm{PQ}}}\left(-18\,\cos^2\alpha\,\sin\alpha\,x + 9\,\sin^3\alpha\,x + 3\,\cos^3\alpha\,x^2\right. \\
&\quad \left.\left. -\cos\alpha\,\sin^2\alpha(-9+x^2)\right)\right\} F_1^2 F_2 \equiv \frac{A_{F_2F_1F_1}}{2}F_1^2 F_2
\end{aligned}
\tag{39}
$$

For the full trilinear $F_2\psi'\psi'$ interaction, we have to add up the terms in Eqs. (34) and (38) to obtain

$$
\begin{aligned}
L_{F_2\psi'\psi'} &= \frac{M_{F_2}^2}{2x\sqrt{9+x^2}F_{\mathrm{PQ}}}\left(-(3\sin\alpha + x\cos\alpha)\left(x^2+9\right)\frac{M_{\psi'}^2}{M_{F_2}^2} + \right. \\
&\quad \left.\left(9\cos\alpha + \sin\alpha x^3\right)\left(1-2\frac{M_{\psi'}^2}{M_{F_2}^2}\right)\right)F_2\psi'\psi' \equiv \frac{A_{F_2\psi'\psi'}}{2}\psi'^2 F_2
\end{aligned}
\tag{40}
$$

Collecting the above formulae one finds the decay rates among flatons and axions

$$
\Gamma\left(F_2 \to aa\right) = \frac{1}{32\pi}\frac{M_{F_2}^3}{F_{\mathrm{PQ}}^2\left(x^2+9\right)}\left(x\cos\alpha + 9\sin\alpha\right)^2
$$

$$
\Gamma\left(F_1 \to aa\right) = \frac{1}{32\pi}\frac{M_{F_1}^3}{F_{\mathrm{PQ}}^2\left(x^2+9\right)}\left(-x\sin\alpha + 9\cos\alpha\right)^2
$$

$$
\Gamma\left(F_2 \to F_1F_1\right) = \frac{1}{32\pi M_{F_2}}\sqrt{1-4\frac{M_{F_1}^2}{M_{F_2}^2}}|A_{F_2F_1F_1}|^2
$$

$$
\Gamma\left(F_2 \to \psi'\psi'\right) = \frac{1}{32\pi M_{F_2}}\sqrt{1-4\frac{M_{\psi'}^2}{M_{F_2}^2}}|A_{F_2\psi'\psi'}|^2
\tag{41}
$$

$$
\Gamma\left(F_2 \to a\psi'\right) = \frac{1}{16\pi}\frac{M_{F_2}^3}{F_{\mathrm{PQ}}^2\left(x^2+9\right)}\left(1-\frac{M_{\psi'}^2}{M_{F_2}^2}\right)^3\left(3\cos\alpha - 3x\sin\alpha\right)^2
$$

$$
\Gamma\left(\psi' \to aF_2\right) = \Gamma\left(F_2 \to a\psi'\right)\Big|_{M_{\psi'}^2 \leftrightarrow M_{F_2}^2}
$$

$$
\Gamma\left(\psi' \to aF_1\right) = \Gamma\left(F_1 \to a\psi'\right)\Big|_{M_{\psi'}^2 \leftrightarrow M_{F_1}^2}
$$

Energy conservation will of course forbid some of these reactions, depending on the flaton masses. As $M_{F_1} < M_{\psi'}$ the channels $F_1 \to \psi'\psi'$ and $F_1 \to \psi'a$ are always forbidden.

4.2 Interaction terms between flatons and flatinos

The trilinear Lagrangian terms responsible for the decay of flatons or flatinos are

$$
L_{\phi\bar{\tilde\phi}\tilde\phi} = \frac{3f}{2M_{\mathrm{Pl}}}\left((v_P Q + v_Q P)\,\bar{\tilde P}\tilde P + i\frac{v_P^2 + 3v_Q^2}{F_{\mathrm{PQ}}}\psi'\,\bar{\tilde P}\gamma_5\tilde P - i2\frac{v_P v_Q}{F_{\mathrm{PQ}}}a\bar{\tilde P}\gamma_5\tilde P\right) +
$$

$$\frac{3f}{2M_{\text{Pl}}}\left(2Pv_P\bar{\tilde{P}}\tilde{Q} + 2i\frac{v_P}{F_{\text{PQ}}}\left(3v_Q\psi' + v_Pa\right)\bar{\tilde{P}}\gamma_5\tilde{Q}\right) \tag{42}$$

(the tilded fields are the fermionic superpartner of the respective P and Q scalars). Let us denote the Yukawa couplings between the flaton (or the axion) and the flatinos in mass basis by $-L_{Yuk} = Y_{ijk}\phi_i\bar{\tilde{F}}_j\left(1, \gamma_5\right)\tilde{F}_k/2$ where γ_5 is taken for $\phi_i = a, \psi'$. We find from Eq. (42) the following expressions for the Yukawa couplings

$$Y_{F_1\tilde{F}_1\tilde{F}_1} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[(x\cos\alpha - \sin\alpha)\cos^2\tilde{\alpha} - x\sin\alpha\sin 2\tilde{\alpha}]$$

$$Y_{F_1\tilde{F}_2\tilde{F}_2} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[(x\cos\alpha - \sin\alpha)\sin^2\tilde{\alpha} + x\sin\alpha\sin 2\tilde{\alpha}]$$

$$Y_{F_1\tilde{F}_1\tilde{F}_2} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[-x\sin\alpha\cos 2\tilde{\alpha} + \frac{1}{2}\left(\sin\alpha - x\cos\alpha\right)\sin 2\tilde{\alpha}]$$

$$Y_{F_2\tilde{F}_1\tilde{F}_1} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[(x\sin\alpha + \cos\alpha)\cos^2\tilde{\alpha} + x\cos\alpha\sin 2\tilde{\alpha}]$$

$$Y_{F_2\tilde{F}_2\tilde{F}_2} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[(x\sin\alpha + \cos\alpha)\sin^2\tilde{\alpha} - x\cos\alpha\sin 2\tilde{\alpha}]$$

$$Y_{F_2\tilde{F}_1\tilde{F}_2} = \frac{6f\mu\sqrt{x^2+9}}{gxF_{\text{PQ}}}[+x\cos\alpha\cos 2\tilde{\alpha} - \frac{1}{2}\left(\cos\alpha + x\sin\alpha\right)\sin 2\tilde{\alpha}]$$

$$Y_{\psi'\tilde{F}_1\tilde{F}_1} = \frac{6f\mu}{gxF_{\text{PQ}}}[-\left(3+x^2\right)\cos^2\tilde{\alpha} - 3x\sin 2\tilde{\alpha}] \tag{43}$$

$$Y_{\psi'\tilde{F}_2\tilde{F}_2} = \frac{6f\mu}{gxF_{\text{PQ}}}[-\left(3+x^2\right)\sin^2\tilde{\alpha} + 3x\sin 2\tilde{\alpha}]$$

$$Y_{\psi'\tilde{F}_1\tilde{F}_2} = \frac{6f\mu}{gxF_{\text{PQ}}}[-3x\cos 2\tilde{\alpha} + \frac{1}{2}\left(3+x^2\right)\sin 2\tilde{\alpha}]$$

$$Y_{a\tilde{F}_1\tilde{F}_1} = \frac{6f\mu}{gxF_{\text{PQ}}}[2x\cos^2\tilde{\alpha} - x^2\sin 2\tilde{\alpha}]$$

$$Y_{a\tilde{F}_2\tilde{F}_2} = \frac{6f\mu}{gxF_{\text{PQ}}}[2x\sin^2\tilde{\alpha} + x^2\sin 2\tilde{\alpha}]$$

$$Y_{a\tilde{F}_1\tilde{F}_2} = \frac{6f\mu}{gxF_{\text{PQ}}}[-x^2\cos 2\tilde{\alpha} - x\sin 2\tilde{\alpha}]$$

From this we can extract the decay rates for $F_i \to \tilde{F}_j\tilde{F}_k$, $\psi' \to \tilde{F}_j\tilde{F}_k$, or $\tilde{F}_2 \to \tilde{F}_1F_i$ (ψ', a)

$$\Gamma\left(F_i \to \tilde{F}_j\tilde{F}_k\right) = \frac{M_{F_i}}{8\pi}S\left(1 - \frac{(M_{\tilde{F}_j} + M_{\tilde{F}_k})^2}{M_{F_i}^2}\right)^{\frac{3}{2}}\left(1 - \frac{(M_{\tilde{F}_j} - M_{\tilde{F}_k})^2}{M_{F_i}^2}\right)^{\frac{1}{2}}Y_{F_i\tilde{F}_j\tilde{F}_k}^2$$

$$\Gamma\left(\psi' \to \tilde{F}_j\tilde{F}_k\right) = \frac{M_{\psi'}}{8\pi}S\left(1 - \frac{(M_{\tilde{F}_j} + M_{\tilde{F}_k})^2}{M_{\psi'}^2}\right)^{\frac{1}{2}}\left(1 - \frac{(M_{\tilde{F}_j} - M_{\tilde{F}_k})^2}{M_{\psi'}^2}\right)^{\frac{3}{2}}Y_{\psi'\tilde{F}_j\tilde{F}_k}^2 \tag{44}$$

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$$\Gamma\left(\tilde{F}_2 \to \tilde{F}_1 F_i\right) = \frac{M_{\tilde{F}_2}}{16\pi}\left(1 - \frac{(M_{\tilde{F}_1} + M_{F_i})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(1 - \frac{(M_{\tilde{F}_1} - M_{F_i})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(\left(1 + \frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2 - \frac{M_{F_i}^2}{M_{\tilde{F}_2}^2}\right)Y_{F_i\tilde{F}_1\tilde{F}_2}^2$$

$$\Gamma\left(\tilde{F}_2 \to \tilde{F}_1 \psi'\right) = \frac{M_{\tilde{F}_2}}{16\pi}\left(1 - \frac{(M_{\tilde{F}_1} + M_{\psi'})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(1 - \frac{(M_{\tilde{F}_1} - M_{\psi'})^2}{M_{\tilde{F}_2}^2}\right)^{\frac{1}{2}}\left(\left(1 - \frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2 - \frac{M_{\psi'}^2}{M_{\tilde{F}_2}^2}\right)Y_{\psi'\tilde{F}_1\tilde{F}_2}^2$$

$$\Gamma\left(\tilde{F}_2 \to \tilde{F}_1 a\right) = \frac{M_{\tilde{F}_2}}{16\pi}\left(1 - \frac{M_{\tilde{F}_1}^2}{M_{\tilde{F}_2}^2}\right)\left(1 - \frac{M_{\tilde{F}_1}}{M_{\tilde{F}_2}}\right)^2 Y_{a\tilde{F}_1\tilde{F}_2}^2$$

where S is a symmetric factor (1/2 for identical final states or otherwise 1).

5 Interaction of flatons and flatinos with matter fields

Now we study the interactions of the flatons with matter and supermatter, specified by Eq. (7) for the KSVZ case and by Eq. (8) for the DSFZ case. Through these interactions the over-produced flatons or flatinos could decay into ordinary matter while the number of the decay produced axions are sufficiently suppressed satisfying the nucleosynthesis limit (10).

5.1 KSVZ model: Interactions between flatons and gluons

We begin with the hadronic models in which the only decay mode available for the flatons is into two gluons coming from the anomaly (when the space phase will be available, we have to take into account also the decay into massive gluinos, in this discussion we neglect such a possibility). The respective one loop corrected decay rates are

$$\Gamma\left(F_1 \to g + g\right) = \frac{\alpha_S^2\left(M_{F_1}\right)}{72\pi^3}N_E^2\frac{M_{F_1}^3}{x^2\,F_{\text{PQ}}^2}\left(x^2 + 9\right)\sin^2\alpha\left(1 + \frac{95}{4}\frac{\alpha_S\left(M_{F_1}\right)}{\pi}\right) \qquad (45)$$

and

$$\Gamma\left(F_2 \to g + g\right) = \frac{\alpha_S^2\left(M_{F_2}\right)}{72\pi^3}N_E^2\frac{M_{F_2}^3}{x^2\,F_{\text{PQ}}^2}\left(x^2 + 9\right)\cos^2\alpha\left(1 + \frac{95}{4}\frac{\alpha_S\left(M_{F_2}\right)}{\pi}\right) \qquad (46)$$

where N_E is the total number of the superheavy exotic quark fields ($M_E = h_E v_P \gg M_{F_i}$).

We do not consider the flatino decay into a gluon and a gluino which will be irrelevant for our discussion.

5.2 DFSZ model: Interactions between flatons/flatinos and ordinary matter

The decay properties of the flatons in the DFSZ models involves the direct interactions between flatons and ordinary matter and supermatter In general the interaction between flatons and Higgs fields are quite interesting due to the fact that this two sectors, after the spontaneous breaking of the PQ and the EW symmetry, mix together. We notice that the

Peccei-Quinn symmetry prevents the introduction of a SUSY invariant mass term $\mu H_1 H_2$, solving automatically the so called μ mass problem as mentioned before.

Let us start by writing the Higgs-flaton potential

$$
\begin{aligned}
V(H,\phi) &= |H_1|^2 \left(m_{H_1}^2 + \left| g \frac{\phi_P \phi_Q}{M_{\mathrm{Pl}}} \right|^2 \right) + |H_2|^2 \left(m_{H_2}^2 + \left| g \frac{\phi_P \phi_Q}{M_{\mathrm{Pl}}} \right|^2 \right) \\
&+ \left\{ g H_1 H_2 \left(A_g \frac{\phi_P \phi_Q}{M_{\mathrm{Pl}}} + 3 f^* \frac{\phi_P^{*2} |\phi_Q|^2}{M_{\mathrm{Pl}}^2} + f^* \frac{\phi_P^{*2} |\phi_P|^2}{M_{\mathrm{Pl}}^2} \right) + \mathrm{c.c.} \right\} \\
&+ \frac{1}{8} \left(g^2 + g'^2 \right) \left(|H_1|^2 - |H_2|^2 \right)^2 .
\end{aligned}
\tag{47}
$$

When the fields $\phi_{P,Q}$ get vevs, the $m_3^2 H_1 H_2$ mass term is generated dynamically. The size of such a term is fixed by

$$
m_3^2 = \mu \left(A_g + \frac{f}{g} \mu \left(x^2 + 3 \right) \right)
\tag{48}
$$

In the limit $|m_3^2| \gg M_W^2$ the masses of the pseudoscalar A^0, of the CP even scalar Higgs field H^0 and of the charged Higgs fields $H^\pm$ are almost degenerate

$$
m_{A^0,H^0,H^\pm}^2 \simeq - \frac{m_3^2}{\sin\beta \cos\beta}
\tag{49}
$$

so from the constraint of positivity of such a masses we get

$$
\frac{A_g}{\mu} + \frac{f}{g}(x^2 + 3) \leq 0
\tag{50}
$$

In such a limit we also know that the mass eigenstate of the CP even electroweak sector H^0, h^0 and of the CP odd one A^0, G^0 are

$$
\begin{aligned}
H^0 &= -\sin\beta \, h_1^0 + \cos\beta \, h_2^0 \\
h^0 &= \cos\beta \, h_1^0 + \sin\beta \, h_2^0 \\
A^0 &= \sin\beta \, A_1^0 + \cos\beta \, A_2^0 \\
G^0 &= \cos\beta \, A_1^0 - \sin\beta \, A_2^0
\end{aligned}
\tag{51}
$$

where $H_1 = \frac{1}{\sqrt{2}} (v_1 + h_1^0 + i A_1^0)$ and $H_2 = \frac{1}{\sqrt{2}} (v_2 + h_2^0 + i A_2^0)$ are the gauge eigenstates and $\tan\beta = v_2/v_1$. To allow the flaton decay into A^0, we want it to be light so that small $\tan\beta$ is preferred in our discussion. Hereafter we will take $\tan\beta = 1$.

From Eq. (48), we find

$$
V_{Fhh} = \frac{1}{2}\mu^2 \left(h_1^{02} + h_2^{02} + A_1^{02} + A_2^{02} \right) \left(\frac{P}{v_P} + \frac{Q}{v_Q} \right) + \frac{1}{2} \left[\left(h_1^0 h_2^0 - A_1^0 A_2^0 \right) + i \left(h_1^0 A_2^0 + h_2^0 A_1^0 \right) \right]
\tag{52}
$$

$$
\left[A_g \mu \left(\frac{P}{v_P} + \frac{Q}{v_Q} - i\frac{x^2+3}{xF_{\mathrm{PQ}}}\psi' \right) + 6\frac{f}{g}\mu^2 \left(\frac{P}{v_P} + \frac{Q}{v_Q} + i\frac{3}{xF_{\mathrm{PQ}}}\psi' \right) + x^2\frac{f}{g}\mu^2 \left(4\frac{P}{v_P} + i\frac{6}{xF_{\mathrm{PQ}}}\psi' \right) \right]
$$

$$
+ \mathrm{c.c.}
$$

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It is then simple manner to get the decay rates for the kinematically more favorable decay channels $F_{1,2} \to h^0 h^0$ and $\psi \to h^0 A^0$

$$\Gamma\left(F_1 \to h^0 h^0\right) = \frac{M_{F_1}^3}{32\pi F_{\mathrm{PQ}}^2} \frac{(x^2+9)}{16\,x^2} \frac{\mu^4}{M_{F_1}^4} \left(1 - \frac{4M_{h^0}^2}{M_{F_1}^2}\right)^{1/2} |A_{F_1 hh}|^2$$

$$\Gamma\left(F_2 \to h^0 h^0\right) = \frac{M_{F_2}^3}{32\pi F_{\mathrm{PQ}}^2} \frac{(x^2+9)}{16\,x^2} \frac{\mu^4}{M_{F_2}^4} \left(1 - \frac{4M_{h^0}^2}{M_{F_2}^2}\right)^{1/2} |A_{F_2 hh}|^2 \quad (53)$$

$$\Gamma\left(\psi' \to h^0 A^0\right) = \frac{M_{\psi'}^3}{16\pi F_{\mathrm{PQ}}^2} \frac{\mu^4}{M_{\psi'}^4} \left(1 - \frac{(M_{h^0}-M_{A^0})^2}{M_{\psi'}^2}\right)^{1/2} \left(1 - \frac{(M_{h^0}+M_{A^0})^2}{M_{\psi'}^2}\right)^{1/2} |A_{\psi h A}|^2$$

where

$$A_{F_1 hh} = \sin 2\beta [\left(\frac{A_g}{\mu} + 6\frac{f}{g}\right)(x\cos\alpha - \sin\alpha) - 4x^2\frac{f}{g}\sin\alpha] + 2(x\cos\alpha - \sin\alpha)$$

$$A_{F_2 hh} = \sin 2\beta [\left(\frac{A_g}{\mu} + 6\frac{f}{g}\right)(x\sin\alpha + \cos\alpha) + 4x^2\frac{f}{g}\cos\alpha] + 2(x\sin\alpha + \cos\alpha)$$

$$A_{\psi' hA} = \left(\frac{A_g}{\mu} - 6\frac{f}{g}\right)\frac{(x^2+3)}{x} \quad (54)$$

If flatons produce a large number of flatinos, flatino decay into axions has to has to be suppressed as well. Primary importance is the production of the lightest flatino $\tilde{F}_1$ which cannot decay into other flatons (axions) or flatinos. The flatino decay into ordinary particles comes from the superpotential $W = \frac{g}{M_{\mathrm{Pl}}}\hat{H}_1\hat{H}_2\hat{P}\hat{Q}$. We find that the flatino decay into a Higgs and a Higgsino (more precisely, the lightest neutralino χ_1) has the rate;

$$\Gamma\left(\tilde{F}_i \to \chi_1 h^0\right) = \frac{M_{\tilde{F}_i}^3}{8\pi F_{\mathrm{PQ}}^2} \frac{\mu^2}{M_{\tilde{F}_i}^2} \left(x^2+9\right)^2 C_{\tilde{F}_i}^2 \left(1 - \frac{(M_{\chi_1}+M_{h^0})^2}{M_{\tilde{F}_i}^2}\right)^{1/2} \left(\left(1 + \frac{M_{\chi_1}}{M_{\tilde{F}_i}}\right)^2 - \frac{M_{h^0}^2}{M_{\tilde{F}_i}^2}\right)$$

where $C_{\tilde{F}_1} = (\sin\tilde{\alpha} + x^{-1}\cos\tilde{\alpha})N_{\chi_1}$ and $C_{\tilde{F}_2} = (\cos\tilde{\alpha} - x^{-1}\sin\tilde{\alpha})N_{\chi_1}$. Here N_{χ_1} denotes the fraction of lightest neutralino in Higgsinos.

Let us now consider the flaton decay into ordinary fermions or sfermions. The mixing terms between flaton and Higgs fields allow a direct tree level coupling (after full mass matrix diagonalization) between the usual fermions and flatons. Parameterizing such a mixing with the parameter θ_{FH} the effective flaton-fermion interaction is $h_f\theta_{FH}$ so that the rate of decay is

$$\Gamma\left(F_i \to f + \bar{f}\right) = N_c \frac{h_f^2\theta_{FH}^2}{16\pi} M_{F_i} \sqrt{\left(1 - 4\frac{m_f^2}{M_{F_i}^2}\right)^3} \quad (55)$$

where N_c is a color factor for the fermion f. Since $\theta_{FH} \simeq \left(\frac{v_{EW}}{F_{PQ}}\right)$, for light fermions $(2m_f < M_F)$ $\Gamma\left(F_i \to f + \bar{f}\right)/\Gamma\left(F_i \to a + a\right) \sim h_f^2 v_{EW}^2/M_{F_i}^2 \sim m_f^2/M_{F_i}^2$ which is less than one. Therefore, the rate of the flaton decay into ordinary fermion cannot be made sufficiently larger than that into axions.

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For the coupling between sfermions and flatons, we have two contributions. One is a direct coupling coming from the scalar potential

$$V_{F\tilde{f}\tilde{f}} = \frac{\mu}{F_{\mathrm{PQ}}} \frac{\sqrt{x^2+9}}{x} \frac{v_1}{\sqrt{2}} \left(h_d \tan\beta \tilde{D}_L^{*}\tilde{D}_R^{*} + h_e \tan\beta \tilde{E}_L^{*}\tilde{E}_R^{*} + h_u \tilde{U}_L^{*}\tilde{U}_R^{*} \right) \quad (56)$$
$$(F_1 \left(x\cos\alpha - \sin\alpha \right) + F_2 \left(\cos\alpha + x\sin\alpha \right)) + h.c.$$

where $\tilde{D}^{*}$ denote down-type squark, *etc.*

The other arises from an indirect coupling induced by the mixing between Higgs and flaton fields as for the fermion case. Taking in consideration the cubic soft A-terms we find

$$V_{eff} = h_d A_d \, \theta_{F_i H_1} \, F_i \tilde{D}_L \tilde{D}_R + h_e A_e \, \theta_{F_i H_1} \, F_i \tilde{E}_L \tilde{E}_R + h_u A_u \, \theta_{F_i H_2} \, F_i \tilde{U}_L \tilde{U}_R \quad (57)$$

so that effectively we have couplings of the size

$$G_{F\,sfermion} \sim h_f(\mu + A_f) \frac{v_{\mathrm{EW}}}{F_{\mathrm{PQ}}} \quad (58)$$

Diagonalizing the sfermion mass matrix we can write $\tilde{f}_R \tilde{f}_L = a_{11}\tilde{f}_1\tilde{f}_1 + a_{22}\tilde{f}_2\tilde{f}_2 + a_{12}\tilde{f}_1\tilde{f}_2$ (where $a_{ii} \propto h_f$ so for $h_f \to 0$ we have $a_{12} \to 1$). Considering the decay of the light flaton we get

$$\Gamma\left(F_1 \to \tilde{f}_i + \tilde{f}_j \right) \simeq N_c \frac{G_{F\tilde{f}}^2}{64\pi \, M_{F_1}} a_{ij}^2 \sqrt{1 - 4\frac{m_{\tilde{f}}^2}{M_{F_1}^2}} \quad (59)$$

As observed in Ref. [15], the flaton may decay efficiently to two light stops as $h_t \sim 1$ and $a_{ij} \sim 1$ and thus a large splitting between light and heavy stops helps increasing the flaton decay rate to light stops. This kind of mass splitting occurs also in the Higgs sector and furthermore the light Higgs (h^o) is usually substantially lighter than the heavy Higgs (H^o) in the minimal supersymmetric standard model. This should be contrasted to the case with the mass splitting for stops which requires some adjustment in soft parameters. In this paper, therefore, we will concentrate on the flaton decay into Higgses as a dominant mode of flaton decays.

6 Parameter space analysis

Our task is now to find the parameter space for which B_a gets small enough. As a reference, let us try to see if $B_a < 0.1$ (corresponding to $\delta N_\nu < 0.7$) can be obtained imposing the stronger condition that $B_I < 0.1$ for each $I = F_{1,2}, \psi'$. We first note that the decay rates calculated in the previous sections are functions of the 4 variables $x = v_P/v_Q$, f/g, A_f/μ and A_g/μ disregarding their overall dependence on F_{PQ}.

Before starting our discussion, let us make some remarks. We are dealing with two kind of PQ models with a natural intermediate scale 1) the DFSZ and 2) the Hadronic model (KSVZ).

i) They have a common flaton potential and thus the same flaton and flatino spectrum. But they have different interactions between flatons and matter.

ii) The symmetries and parameter space constraints impose that the following decays are forbidden $\psi' \to aa$, $F_1 \to \psi'a$, $F_1 \to \psi'\psi'$, $F_1 \to \tilde{F}\tilde{F}$.

Neglecting for the moment the model dependent flaton-matter interactions, we have to analyze the decays $F_{1,2} \to a\,a$, $F_2 \to \psi'a$ and the orthogonal $\psi' \to F_{1,2}\,a$ plus the flatons-flatinos interactions that through the chain $Flaton \to Flatinos \to Flatino - axion$ can also generate a non negligible axionic density at nucleosynthesis time.

The decays of the flatons into two axions doesn't have any phase space suppression. If we choose $\alpha = \alpha_1$ (see below Eq. 22) then we can suppress the rate $\Gamma(F_1 \to a + a)$ only taking $x^2 = 18$ and $\cos\alpha_1|_{x^2=18} = 0.426$ (giving for example $\Gamma(F_2 \to a + a) = 3.6\frac{M_{F_2}^3}{32\pi F_{\rm PQ}^2}$). On the other hand, if we choose $\alpha = \alpha_2$ we can suppress only $\Gamma(F_2 \to a + a)$ in the region $x^2 = 18$ and $\cos\alpha_2|_{x^2=18} = -0.905$ (giving for example $\Gamma(F_2 \to a + a) = 3.6\frac{M_{F_1}^3}{32\pi F_{\rm PQ}^2}$).

The decay rate into a single axion has the phase space suppression constraint $m_{F_2}^2 \geq M_{\psi'}^2$ that translated in our parameters reads $-\frac{g}{f}\frac{A_f}{\mu} \leq 12$.

When the flaton's branching ratio to flatinos becomes sizable we have also to impose

$$B\left(I \to \tilde{F}_2\right) B\left(\tilde{F}_2 \to a\tilde{F}_1\right) < 0.1\,. \tag{60}$$

with $B\left(I \to \tilde{F}_2\right) = 2\,B\left(F_2 \to \tilde{F}_2\tilde{F}_2\right) + B\left(F_2 \to \tilde{F}_2\tilde{F}_1\right) + 2\,B\left(\psi' \to \tilde{F}_2\tilde{F}_2\right) + B\left(\psi' \to \tilde{F}_2\tilde{F}_1\right)$ and also the important requirement that $\tilde{F}_1$ has to be heavier than the lightest neutralino (χ_1^0) and the light Higgs (h^o) since it has the unique decay mode to a neutralino and a Higgs. Assuming $m_{\tilde{F}_1}$ is much larger than the light Higgs mass, we impose a strong condition $m_{\tilde{F}_1} > \mu$ that translates into the approximate bound $\left|\frac{f}{g}\right| > \frac{1}{3x}$.

6.1 Parameter space of KSVZ models

As discussed already, the light flaton F_1 can decay only into two gluons, so the simple requirement $B_{F_1}^{-1} = \Gamma[F_1 \to gg]/\Gamma[F_1 \to aa] \gg 1$ impose a quite strong constraint on the parameter space. This ratio is given by

$$\frac{\Gamma(F_1 \to g + g)}{\Gamma(F_1 \to a + a)} = \frac{\alpha_S^2(M_{F_1})}{\pi^2}\frac{4}{9}N_E^2\left(\frac{\sin\alpha(x^2 + 9)}{x(9\cos\alpha - x\sin\alpha)}\right)^2\left(1 + \frac{95}{4}\frac{\alpha_S(M_{F_1})}{\pi}\right)\,. \tag{61}$$

In order to have a large number we can use only the x parameter and the number of exotic quark (to be as large as possible). For $\alpha_S(M_F) \simeq 0.1$, this ratio can be larger than 10 accepting a tuning of the x parameter as follows

$$\begin{aligned}
\sqrt{18} - 0.04 < \quad x \quad < \sqrt{18} + 0.04 \quad \text{for} \quad N_E = 1, \\
\sqrt{18} - 0.3 < \quad x \quad < \sqrt{18} + 0.3 \quad \text{for} \quad N_E = 9.
\end{aligned} \tag{62}$$

However, in such region we don't find any solution in which, $B_a(F_2)$ and $B_a(\psi')$ are less than 0.1 at same time. Therefore the KSVZ model cannot give a satisfactory solution of the

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nucleosynthesis bound unless some extra fine tuning on the initial densities of the flatons F_2, ψ' can be dynamically achieved.

6.2   Parameter space of DFSZ models

In the DFSZ model the number of the possible decay channels is much larger than the other model and it is not an easy task to find, in the four parameter space, some easy understandable avaliable region. To be as independent as possible of the soft supersymmetry breaking parameters we will try to make analytic computations on the rates of the flaton decays into Higgs particles, in particular into the lightest Higgs (h^0) whose mass has an automatic upper bound of ~ 140 GeV [20]. We will concentrate on the region with $\frac{f}{g}$ negative and $\left|\frac{f}{g}\right| \ll 1$ and $x \gg 1$ which turns out to be required for $B_a < 0.1$.

To open the decay channels of the flatons into Higgs particles we have, in particular, to impose $M_{\psi'} > M_A > 0$ which requires (from now on we will use the unequal symbols as strictly satisfied) $\frac{A_g}{\mu} < \left|\frac{f}{g}\right| x^2$ with

$$\left|\frac{f}{g}\frac{A_f}{\mu}\right| x^2 > 2 \left|\frac{A_g}{\mu} + \frac{f}{g} x^2\right|. \tag{63}$$

For simplicity we divide into two regimes

$$|\frac{A_g}{\mu}| < |\frac{f}{g}| x^2, \qquad \left|\frac{A_f}{\mu}\right| > 2 \tag{64}$$

and

$$\left|\frac{A_g}{\mu}\right| > \left|\frac{f}{g}\right| x^2, \qquad \left|\frac{A_g}{A_f}\right| < \frac{1}{2} x^2 \left|\frac{f}{g}\right|, \qquad \left|\frac{A_f}{\mu}\right| > 2. \tag{65}$$

Remember that the positivity of flaton masses requires

$$\left|\frac{A_f}{\mu}\right| < x^2 \left|\frac{f}{g}\right|. \tag{66}$$

Combining altogether we get $\left|\frac{f}{g}\right| > \frac{1}{2x^2}$ in the regime (64) and $\left|\frac{f}{g}\right| > \frac{2}{x^2}$ in the regime (65). Besides, if flatino production rates are sizable, we also have to impose $R_{\tilde{F}_2} \equiv \Gamma(\tilde{F}_2 \to \chi_1 h^0)/\Gamma(\tilde{F}_2 \to a\tilde{F}_1) > 10$ and $M_{\tilde{F}_1} > M_{\chi_1} + M_{h^0}$ to open the decay mode $\tilde{F}_1 \to \chi_1 h^0$. The condition $R_{\tilde{F}_2} > 10$ gives the restriction $|f/g| < 0.02\, x\, N_{\chi_1}$ and $|\frac{f}{g}| > \frac{1}{3x}$.

Then we study, in our limit, the constraints given by the conditions $R_{\psi'} \equiv \Gamma(\psi' \to h^0 A^0)/\Gamma(\psi' \to aF_1) > 10$ and $R_{F_i} \equiv \Gamma(F_i \to\to h^0 h^0)/\Gamma(F_i \to aa) > 10$. The ratios R are

$$R_{\psi'} \sim \frac{1}{144} \left(\frac{g}{f}\right)^2 \frac{\mu^2}{A_f^2} \left(\frac{A_g}{\mu} - 6\frac{f}{g}\right)^2$$

$$R_{F_1} \sim \frac{1}{4} \frac{\mu^4}{M_{\tilde{F}_1}^4} (\frac{A_g}{\mu} - 2\frac{f}{g} x^2 + 2)^2 \tag{67}$$

$$R_{F_2} \sim 10^{-3} x^4 \frac{\mu^4}{M_{\tilde{F}_2}^4} (\frac{A_g}{\mu} + 18\frac{f}{g} + 2)^2$$

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where $M_{F_1}^2 \sim |\frac{f}{g}\frac{A_f}{\mu}|x^2\mu^2$ and $M_{F_2}^2 \sim 12\frac{f^2}{g^2}x^2\mu^2$ for $\frac{A_f}{\mu} < 12|\frac{f}{g}|$, and $M_{F_1} \leftrightarrow M_{F_2}$ for $\frac{A_f}{\mu} > 12|\frac{f}{g}|$. We can now divide the parameter space into four regions.

I) $\left|\frac{A_g}{\mu}\right| > \left|\frac{f}{g}\right|x^2$, II) $2 < \left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right|x^2$, III) $\left|\frac{f}{g}\right| < \left|\frac{A_g}{\mu}\right| < 2$, IV) $\left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right|$.

Depending on $\frac{A_f}{\mu} < 12|\frac{f}{g}|$ or $\frac{A_f}{\mu} > 12|\frac{f}{g}|$ we will have region a or region b.

We find that all of the a regions are forbidden, and so is the IV_b region. The constraints for the regions are as follows.

$$
\begin{aligned}
I_b \quad & x > 14, \quad A_g < 0, \quad 12\left|\frac{f}{g}\right| < \left|\frac{A_f}{\mu}\right| \\
& 1 < \frac{1}{2}\left|\frac{A_f}{\mu}\right| < \left|\frac{f}{g}\right|\frac{x^2}{2} < \left|\frac{A_g}{\mu}\right| < 2\left(\left|\frac{f}{g}\right|\frac{x^2}{2}\right)^2 . \\
II_b \quad & x > 9, \quad A_g > 0, \quad \left|\frac{f}{g}\right| < 3\times 10^{-2} \\
& 2 < \left|\frac{A_f}{\mu}\right| < \left|\frac{f}{g}\right|x^2, \quad 2 < \left|\frac{A_g}{\mu}\right| < \left|\frac{f}{g}\right|x^2 . \\
III_b \quad & A_g > 0, \quad 1 < 2\left|\frac{f}{g}\right|x^2, \\
& 2\left|\frac{f}{g}\right| < \left|\frac{A_f}{\mu}\right|\left|\frac{f}{g}\right| < 2\times 10^{-2}, \quad \left|\frac{f}{g}\right| < \left|\frac{A_g}{\mu}\right| < 2 .
\end{aligned}
\tag{68}
$$

To summarize, we find that:
- In all cases, x has to be large ($\gtrsim 10$)
- In cases II and III, $\left|\frac{f}{g}\right|$ has to be small ($< 3\times 10^{-2}$) but it has no useful upper bound in case I. In all cases $x^2\left|\frac{f}{g}\right| \gtrsim 1$.
- In all cases, $|A_f| \sim |A_g| \sim |\mu|$ is a possibility.

7   Conclusions

We have explored the cosmology of a particularly attractive extension of the Standard Model, which has a Peccei-Quinn symmetry broken only by two 'flaton' fields ϕ_P and ϕ_Q, characterized by a very large vev (10^{10-12} GeV) and a relatively small mass (10^{2-3} GeV). These and their superpartners generalize the saxion and axino, that in the non-flat case are the only fields with soft masses.

In contrast with more general models the density of dark matter axions can be estimated with essentially no assumption about other sectors of the theory, assuming only that ϕ_P has a positive effective mass-squared in the early Universe. If ϕ_Q also has a positive effective mass-squared the axion is an excellent dark matter candidate. In the opposite case the axion density is probably to low in this particular model.

Our main concern has been with a different, highly relativistic, axion population that is produced by flaton decay. We have calculated the rates for all relevant channels and examined

the constraint that the energy density of these axions does not upset the predictions of the standard nucleosynthesis.

For the KSVZ model we find that it is almost impossible to satisfy such a cosmological bounds due to the fact that the only flatonic decay channel in competition with the axionic one is into gluons (through axial anomaly) which is too much constrained.

For the DFSZ model there are more decay channels. To evade complicated phase space suppressions we concentrate on the Higgs decay products, as the mass of the lightest Higgs has naturally a relatively low upper bound and the mass of the other Higgs and flaton fields are fixed by the parameters of the flatonic potential itself. We have quantified a portion of the parameter space available showing the strength of this model.

An interesting question, lying beyond the present investigation, is whether the allowed region of parameter space can be achieved in a supergravity model with universal soft parameters.

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